FAR BEYOND

MAT122

Product Rule

Stony Brook University

Review

Power Rule

$$(ax^n)' = \frac{d}{dx}ax^n = nax^{n-1}$$

Exponential Derivative:

(base *e*)

$$\left(e^x\right)' = e^x$$

Special Cases:

$$\frac{d}{dx}x = 1$$

$$\frac{d}{dx}a = 0$$

where a is a constant

Exponent Law:

$$\sqrt[n]{x} = x^{1/n}$$

Do: differentiate $f(x) = 5x^{100} - e^x + 7\sqrt[3]{x} + 11$

Express answer with positive exponents.

$$= \boxed{500x^{99} - e^x + \frac{7}{3x^{2/3}}}$$

Product Rule - Intro

When two *differentiable* functions are multiplied, use the **Product Rule** to take derivative:

shorthand $(f \cdot g)' = f' \cdot g + f \cdot g'$

$$[f(x)g(x)]' = f'(x)\,g(x) + f(x)\,g'(x)$$

Leibniz notation: $$\frac{d}{dx}\left[f(x)g(x)\right] = \frac{d}{dx}f(x)\cdot g(x) + f(x)\cdot\frac{d}{dx}g(x)$$

ex. find the derivative of $h(x) = xe^x$

$g(x) = e^x$

$f(x) = x$

build a chart of functions and their derivatives:

$f(x) = x$	$g(x) = e^x$
$f'(x) = 1$	$g'(x) = e^x$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$= 1 \cdot e^x + x \cdot e^x$$

$$= \boxed{e^x + x\,e^x}$$

Product Rule – cont'd

$$(f\cdot g)' = f'\cdot g + f\cdot g'$$

ex. find the derivative of $h(x) = (x^2+3x)\,(5x^3-2)$

$f(x)$ → (x^2+3x), $g(x)$ → $(5x^3-2)$

$f = x^2+3x$ $\qquad$ $g = 5x^3-2$

$f' = 2x+3$ $\qquad$ $g' = 15x^2$

$$h'(x) = f'g + fg'$$

$$= (2x+3)(5x^3-2) + (x^2+3x)(15x^2)$$

ok to leave in factored form

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ex. differentiate $f(y) = \left(\dfrac{1}{y^2} - 3y^4\right)\left(y+5y^3\right)$

$f = \dfrac{1}{y^2} - 3y^4$ $\qquad$ $g = y+5y^3$

$= y^{-2} - 3y^4$

$f' = -2y^{-3} - 12y^3$ $\qquad$ $g' = 1+15y^2$

$= -\dfrac{2}{y^3} - 12y^3$

$$f'(y) = \left(-\frac{2}{y^3} - 12y^3\right)\left(y+5y^3\right) + \left(\frac{1}{y^2} - 3y^4\right)\left(1+15y^2\right)$$

Product Rule – Do

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

Do: differentiate $f(x) = \left(4x^3 - 6x^2 + 1\right)\left(5x^4 + 7x^2 + 3x\right)$

Do: find $f'(x)$: $f(x) = \left(x + \sqrt{x} + \sqrt[3]{x}\right)\left(e^x - x^2\right)$

Rate of Change - Application

Product Rule

ex. The cost (in dollars) of producing x phone chargers is given by $C(x)=(3x-25)(500-x)$. Find the rate at which cost is changing when 100 chargers have been produced.

derivative

Step 1: find general derivative

$$\text{rate of change} = C'(x) = 3(500-x) + (3x-25)(-1)$$

$f = 3x-25 \qquad g = 500-x$

$f' = 3 \qquad g' = -1$

Step 2: plug 100 into derivative

$$C'(100) = 3(500-100) - (3\cdot 100 - 25)$$

$$= 3(400) \quad - (300-25)$$

$$= 1200 \quad - \quad 275$$

$$= \boxed{\$\,925\text{ /charger}}$$