

**FAR
BEYOND**

MAT122

Product Rule



Stony Brook University

Review

Power Rule

$$(ax^n)' = \frac{d}{dx} ax^n = nax^{n-1}$$

Exponential Derivative:

(base e)

$$(e^x)' = e^x$$

Special Cases:

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} a = 0$$

where a is a constant

Exponent Law:

$$\sqrt[n]{x} = x^{1/n}$$

Do: differentiate $f(x) = 5x^{100} - e^x + 7\sqrt[3]{x} + 11$

Express answer with positive exponents.

$$= 500x^{99} - e^x + \frac{7}{3x^{2/3}}$$

Product Rule - Intro

When two *differentiable* functions are multiplied, use the **Product Rule** to take derivative:

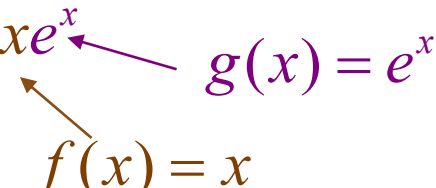
shorthand $(f \cdot g)' = f' \cdot g + f \cdot g'$

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

Leibniz notation:

$$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}f(x) \cdot g(x) + f(x) \cdot \frac{d}{dx}g(x)$$

ex. find the derivative of $h(x) = xe^x$



$f(x) = x$ $g(x) = e^x$

build a chart of functions and their derivatives:

$$f(x) = x$$

$$f'(x) = 1$$

$$g(x) = e^x$$

$$g'(x) = e^x$$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$= 1 \cdot e^x + x \cdot e^x$$

$$= \boxed{e^x + xe^x}$$

Product Rule – cont'd

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

ex. find the derivative of $h(x) = (x^2 + 3x)(5x^3 - 2)$

$f(x)$ $g(x)$

$$f = x^2 + 3x$$

$$f' = 2x + 3$$

$$g = 5x^3 - 2$$

$$g' = 15x^2$$

$$h'(x) = f'g + fg'$$

$$= (2x + 3)(5x^3 - 2) + (x^2 + 3x)(15x^2)$$

ok to leave in factored form

ex. differentiate $f(y) = \left(\frac{1}{y^2} - 3y^4\right)(y + 5y^3)$

$$f'(y) = \left(-\frac{2}{y^3} - 12y^3\right)(y + 5y^3) + \left(\frac{1}{y^2} - 3y^4\right)(1 + 15y^2)$$

$$f = \frac{1}{y^2} - 3y^4$$

$$= y^{-2} - 3y^4$$

$$f' = -2y^{-3} - 12y^3$$

$$= -\frac{2}{y^3} - 12y^3$$

$$g = y + 5y^3$$

$$g' = 1 + 15y^2$$

Product Rule – Do

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$


Do: differentiate $f(x) = (4x^3 - 6x^2 + 1)(5x^4 + 7x^2 + 3x)$

Do: find $f'(x)$: $f(x) = (x + \sqrt{x} + \sqrt[3]{x})(e^x - x^2)$

Rate of Change - Application

Product Rule

ex. The cost (in dollars) of producing x phone chargers is given by $C(x) = (3x - 25)(500 - x)$. Find the rate at which cost is changing when 100 chargers have been produced.

 derivative

Step 1: find general derivative

$$\text{rate of change} = C'(x) = 3(500 - x) + (3x - 25)(-1)$$

$$\begin{array}{ll} f = 3x - 25 & g = 500 - x \\ f' = 3 & g' = -1 \end{array}$$

Step 2: plug 100 into derivative

$$\begin{aligned} C'(100) &= 3(500 - 100) - (3 \cdot 100 - 25) \\ &= 3(400) - (300 - 25) \\ &= 1200 - 275 \\ &= \boxed{\$925 / \text{charger}} \end{aligned}$$